

Non-Newtonian gravity in finite nuclei

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Abstract. In this talk, we report our recent study of constraining the non-Newtonian gravity at femtometer scale. We incorporate the Yukawa-type non-Newtonian gravitational potential consistently to the Skyrme functional form using the exact treatment for the direct contribution and density-matrix expansion method for the exchange contribution. The effects from the non-Newtonian potential on finite nuclei properties are then studied together with a well-tested Skyrme force. Assuming that the framework without non-Newtonian gravity can explain the binding energies and charge radii of medium to heavy nuclei within 2% error, we set an upper limit for the strength of the non-Newtonian gravitational potential at femtometer scale.

1. Introduction

How to understand the nature of the gravitational force and unify it with other fundamental forces are among the most important questions to answer in the new century [1]. It has gradually been realized that the traditional Newtonian gravitational potential, which obeys the Inverse-Square-Law (ISL), can not explain all the phenomena relative to gravity in nature. Great efforts have been devoted to search for the possible existence of non-Newtonian gravity that violates

the ISL [2, 3, 4, 5, 6, 7, 8]. Especially, these studies have put upper limits of the strength of the non-Newtonian gravity from as large as galaxy scale to as small as 10 fm [7]. At galaxy scale, various modified gravity theories are used to study the non-Newtonian gravity such as the scalar-tensor-vector gravity and $f(R)$ gravity. At short distances, the origin of the non-Newtonian potential can be attributed to the possible existence of extra dimensions within string/M theories or exchanging light and weakly-coupled new particles in the supersymmetric extension of the Standard Model.

The following form of the gravitational potential between two objects of masses m_1 and m_2 at positions \vec{r}_1 and \vec{r}_2 has been widely used at both galaxy scale and short distances [9]

$$V_{\text{grav}}(\vec{r}_1, \vec{r}_2) = -\frac{Gm_1m_2}{|\vec{r}_1 - \vec{r}_2|}(1 + \alpha e^{-|\vec{r}_1 - \vec{r}_2|/\lambda}). \quad (1)$$

In the above, the first term is the traditional Newtonian potential where G is gravitational constant, while the second term represents the non-Newtonian potential, where α and λ are the strength and length scale parameters, respectively.

At galaxy scale ($\lambda \sim 10^{21}$ m), this simple Yukawa-type non-Newtonian gravitational potential can be viewed as a reduced form of the scalar-tensor-vector gravity or the $f(R)$ gravity in the weak-field limit [10, 11], and it has been used to successfully explain both the flattening of the galaxy rotation curves away from the Kepler limit [12, 13] and the Bullet Cluster 1E0657-558 observations in the absence of dark matter [14]. For a recent review of applying this Yukawa-type non-Newtonian gravitational potential on well-known observations at galaxy scale, we refer the readers to Ref. [15].

At short distances, the Yukawa potential in Eq. (1) may come from exchanging a light and weakly-coupled spin-0 axion [16] or spin-1 U-boson [17] corresponding respectively to an attractive or repulsive potential. The exchanged boson is related to several interesting new phenomena in particle physics and cosmology [18, 19] and may mediate the annihilation of dark matter particles [20, 21] which accounts for the 511 keV γ -ray from the galactic bulge as well.

Especially, the properties of a neutron star would be affected by the possible existence of the non-Newtonian gravitational potential if its length scale is smaller than the size of the star. In this case, if one still wants to rely on solving the Tolman-Oppenheimer-Volkoff using the equation of state (EOS) of neutron star matter, the above Yukawa-type non-Newtonian gravitational potential can be viewed as an extra potential between nucleons in addition to the basic nucleon-nucleon interaction. It has been found that the possible existence of the repulsive Yukawa potential from exchanging a spin-1 U-boson would stiffen the EOS of neutron star matter compared to that from a nucleon-nucleon interaction only and thus increase both the mass and the radius of a neutron star [22]. Even if with a supersoft symmetry energy at suprasaturation densities, the repulsive Yukawa potential could still maintain a stable neutron star [23]. In addition, the core-crust transition density/pressure could also be affected and the neutron star structure could be modified in the presence of the Yukawa potential [24]. The coupling of U-boson and nucleon has been successfully introduced into the relativistic mean-field model and similar effects have been observed [25]. Furthermore, it is shown that although the presence of hyperons and quarks would soften the EOS of neutron star matter, the repulsive Yukawa potential can stiffen the EOS and account for the heavy mass of PSR J1614-2230 [26]. Thus, our knowledge of the non-Newtonian potential and the nuclear EOS are both important in understanding many interesting phenomena in nuclear astrophysics.

At a certain length scale λ , the non-Newtonian potential is generally compared with the typical interaction for the corresponding length scale, through which the strength of the non-Newtonian potential is constrained if the interaction is well determined. As mentioned above, the study and the constraint of the non-Newtonian potential below 10 fm are still lacking. The most typical system at femtometer scale is a finite nucleus, where the typical interaction is the

strong interaction. Thanks to the great efforts made by the nuclear physicists, the properties of finite nuclei can be very well explained by effective nucleon-nucleon interactions. Especially, from the empirical values of isoscalar and isovector macroscopic quantities, the parameters in the Skyrme effective interaction can be inversely determined, leading to the MSL0 Skyrme force [27]. This gives us the opportunity to study the possible existence of the Yukawa-type non-Newtonian potential in the finite nuclei whose properties have been well described by the MSL0 Skyrme force, and constrain the strength of the non-Newtonian potential at femtometer scale.

2. Incorporating the Yukawa potential to the Skyrme-Hartree-Fock calculation

In the one-boson-exchange picture, the Yukawa-type non-Newtonian gravitational potential between two nucleons at position \vec{r}_1 and \vec{r}_2 is written as

$$V_Y(\vec{r}_1, \vec{r}_2) = \pm \frac{g^2}{4\pi} \frac{e^{-\mu|\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|}. \quad (2)$$

In the above, g is the boson-nucleon coupling constant and it can be expressed as $g = \sqrt{4\pi|\alpha|Gm^2}$ comparing Eq. (2) with Eq. (1), where m is the nucleon mass. $\mu = 1/\lambda$ is the mass of axion or U-boson which is largely quite uncertain, and it determines the length scale of the non-Newtonian potential. The potential in Eq. (2) can be positive or negative, depending on the spin of the exchanged boson. To ease discuss we omit the \pm sign in the following formulas.

To apply the above Yukawa potential to finite nuclei calculation, we need to do Hartree-Fock calculation as for the Skyrme interaction. The additional potential energy due to the existence of the Yukawa potential can thus be written as

$$E_Y = \frac{1}{2} \sum_{i,j} \langle ij | V_Y (1 - P_r P_\sigma P_\tau) | ij \rangle, \quad (3)$$

where P_r , P_σ , and P_τ are the space, spin, and isospin exchange operator, respectively, and $|i\rangle$ is the quantum state of the i th particle containing spatial, spin, and isospin parts. The coefficient $\frac{1}{2}$ is due to the double counting for exchanging i and j .

The first term in Eq. (3) is the direct (Hartree) contribution. By expressing the quantum state in the spatial coordinate representation it is written as

$$E_Y^D = \frac{1}{2} \int \rho(\vec{r}_1) \rho(\vec{r}_2) \frac{g^2}{4\pi} \frac{e^{-\mu|\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|} d^3r_1 d^3r_2, \quad (4)$$

where

$$\rho(\vec{r}) = \sum_{i,\sigma,\tau} \phi_{\tau i}^*(\vec{r}, \sigma) \phi_{\tau i}(\vec{r}, \sigma) \quad (5)$$

is the nucleon number density with $\phi_{\tau i}(\vec{r}, \sigma)$ being the spatial wave function of the i th particle with spin σ and isospin τ .

The second term in Eq. (3) is the exchange (Fock) contribution. By using $P_\sigma = (1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)/2$ where $\vec{\sigma}_{1(2)}$ is the Pauli operator acting on the state $|i\rangle$ ($|j\rangle$), it can be written as

$$E_Y^E = -\frac{1}{4} \sum_{\tau=n,p} \int [\rho_\tau(\vec{r}_1, \vec{r}_2) \rho_\tau(\vec{r}_2, \vec{r}_1) + \vec{\rho}_\tau(\vec{r}_1, \vec{r}_2) \cdot \vec{\rho}_\tau(\vec{r}_2, \vec{r}_1)] \frac{g^2}{4\pi} \frac{e^{-\mu|\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|} d^3r_1 d^3r_2, \quad (6)$$

where

$$\rho_\tau(\vec{r}_1, \vec{r}_2) = \sum_{i,\sigma} \phi_{\tau i}^*(\vec{r}_1, \sigma) \phi_{\tau i}(\vec{r}_2, \sigma) \quad (7)$$

and

$$\vec{\rho}_\tau(\vec{r}_1, \vec{r}_2) = \sum_i \sum_{\sigma, \sigma'} \phi_{\tau i}^*(\vec{r}_1, \sigma') \phi_{\tau i}(\vec{r}_2, \sigma) \langle \sigma' | \vec{\sigma} | \sigma \rangle \quad (8)$$

are the off-diagonal scalar and vector part of the density matrix, respectively.

The finite-range direct contribution can generally be treated exactly, while the finite-range exchange contribution has to be treated using certain approximation. To get similar density functional form as that from the zero-range Skyrme interaction, we use in the following calculation the density-matrix expansion method [28, 29]. By introducing the coordinate transformation $\vec{r} = (\vec{r}_1 + \vec{r}_2)/2$ and $\vec{s} = \vec{r}_1 - \vec{r}_2$, the off-diagonal scalar and vector part of the density matrix can be expanded in the lower orders as

$$\rho_\tau(\vec{r} + \frac{\vec{s}}{2}, \vec{r} - \frac{\vec{s}}{2}) \approx \rho_{SL}(k_\tau s) \rho_\tau(\vec{r}) + g(k_\tau s) s^2 \left[\frac{1}{4} \nabla^2 \rho_\tau(\vec{r}) - \tau_\tau(\vec{r}) + \frac{3}{5} k_\tau^2 \rho_\tau(\vec{r}) \right] \quad (9)$$

and

$$\vec{\rho}_\tau(\vec{r} + \frac{\vec{s}}{2}, \vec{r} - \frac{\vec{s}}{2}) \approx \frac{i}{2} j_0(k_\tau s) \vec{s} \times \vec{J}_\tau(\vec{r}). \quad (10)$$

In the above, $k_\tau = (3\pi^2 \rho_\tau)^{1/3}$ is the Fermi momentum, and τ_τ and \vec{J}_τ are the kinetic density and the spin-current density, which can be respectively expressed as

$$\tau_\tau(\vec{r}) = \sum_{i, \sigma} |\nabla \phi_{\tau i}(\vec{r}, \sigma)|^2, \quad (11)$$

and

$$\vec{J}_\tau(\vec{r}) = -i \sum_i \sum_{\sigma, \sigma'} \phi_{\tau i}^*(\vec{r}, \sigma') \nabla \phi_{\tau i}(\vec{r}, \sigma) \times \langle \sigma' | \vec{\sigma} | \sigma \rangle. \quad (12)$$

The function $\rho_{SL}(k_\tau s)$ and $g(k_\tau s)$ can be expressed respectively as

$$\rho_{SL}(k_\tau s) = \frac{3j_1(k_\tau s)}{k_\tau s}, \quad (13)$$

and

$$g(k_\tau s) = \frac{35j_3(k_\tau s)}{2(k_\tau s)^3}, \quad (14)$$

where j_1 and j_3 are the first- and the third-order spherical Bessel function.

Applying Eqs. (9) and (10) to Eq. (6), a Skyrme-like potential energy density functional form from the exchange contribution of the Yukawa potential can be obtained with density-dependent coefficients as

$$H_Y^E(\vec{r}) = \sum_{\tau=n,p} \{A[\rho_\tau(\vec{r})] + B[\rho_\tau(\vec{r})] \tau_\tau(\vec{r}) + C[\rho_\tau(\vec{r})] [\nabla \rho_\tau(\vec{r})]^2 + \varphi[\rho_\tau(\vec{r})] J_\tau^2(\vec{r})\}, \quad (15)$$

where the expressions of the coefficients are

$$A(\rho_\tau) = -\frac{1}{4} \int \rho_\tau^2 \rho_{SL}^2(k_\tau s) V_Y(s) d^3 s - \frac{3}{5} (3\pi^2)^{2/3} \rho_\tau^{5/3} B(\rho_\tau), \quad (16)$$

$$B(\rho_\tau) = \frac{1}{2} \int \rho_\tau \rho_{SL}(k_\tau s) g(k_\tau s) s^2 V_Y(s) d^3 s, \quad (17)$$

$$C(\rho_\tau) = \frac{1}{4} \frac{dB(\rho_\tau)}{d\rho_\tau}, \quad (18)$$

$$\varphi(\rho_\tau) = -\frac{\pi}{6} \int j_0^2(k_\tau s) s^4 V_Y(s) ds, \quad (19)$$

with $V_Y(s) = g^2 e^{-\mu s} / (4\pi s)$.

In the standard Skyrme-Hartree-Fock calculation, the finite nuclei properties are obtained by solving the following Schrödinger equation

$$\left[-\nabla \cdot \frac{\hbar^2}{2m_\tau^*(\vec{r})} \nabla + U_\tau(\vec{r}) + \vec{W}_\tau(\vec{r}) \cdot (-i\nabla \times \vec{\sigma}) \right] \phi_{\tau i} = e_{\tau i} \phi_{\tau i}, \quad (20)$$

where $e_{\tau i}$ is the eigenvalue of the single-particle energy and $\phi_{\tau i}$ is the nucleon wave function. Due to the existence of the Yukawa-type non-Newtonian gravitational potential, the single-particle potential U_τ , the nucleon effective mass m_τ^* , and the spin-orbit potential \vec{W}_τ are not only determined by the Skyrme interaction but modified by the Yukawa potential as well, and they can be calculated from the variational principle. The modifications of the single-particle potential from the Yukawa potential are from both the direct contribution and the exchange contribution

$$U_\tau^Y = U_\tau^{YD} + U_\tau^{YE}, \quad (21)$$

$$U_\tau^{YD} = \int \rho(\vec{r}') \frac{g^2}{4\pi} \frac{e^{-\mu|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} d^3 r', \quad (22)$$

$$U_\tau^{YE} = \frac{dA(\rho_\tau)}{d\rho_\tau} + \frac{dB(\rho_\tau)}{d\rho_\tau} \tau_\tau - \frac{dC(\rho_\tau)}{d\rho_\tau} (\nabla \rho_\tau)^2 - 2C(\rho_\tau) \nabla^2 \rho_\tau + \frac{d\varphi(\rho_\tau)}{d\rho_\tau} J_\tau^2. \quad (23)$$

We note that the direct contribution is much larger than the exchange contribution. The nucleon effective mass and the spin-orbit potential are modified only by the exchange contribution of the Yukawa potential, respectively, according to

$$\frac{\hbar^2}{2m_\tau^*} \rightarrow \frac{\hbar^2}{2m_\tau^*} + B(\rho_\tau) \quad (24)$$

and

$$\vec{W}_\tau^Y = 2\varphi(\rho_\tau) \vec{J}_\tau. \quad (25)$$

3. Results and discussions

Using the formulism discussed in Sec. 2, we are now able to study the properties of finite nuclei in the presence of the Yukawa-type non-Newtonian gravitational potential. In the following, we will discuss the effects from the Yukawa potential by comparing its strength with the Coulomb interaction in finite nuclei and then set an upper limit of its strength.

3.1. Comparing with the Coulomb interaction

It is instructive to set the strength of the Coulomb interaction as a base line and discuss how the finite nuclei properties will be affected if the non-Newtonian potential in finite nuclei is as strong as the Coulomb potential. In Fig. 1 we compare the Coulomb potential with the non-Newtonian Yukawa potential between two protons with $\alpha = -1.24 \times 10^{36}$ or -1.24×10^{34} , and $\lambda = 1$ fm or 10 fm, respectively. We note that $\alpha = -1.24 \times 10^{36}$ leads to the coupling constant $g^2/4\pi = 1/137$, which is exactly the same strength as the Coulomb potential. Thus, at short distances the Yukawa potential is similar to the Coulomb potential while it decreases faster for a smaller value of λ due to the exponential decay term. The Yukawa potential with only 1% strength of that is also shown for reference.

Next, let's see what would happen in finite nuclei if the Yukawa potential is as strong as the Coulomb potential. It is seen in Fig. 2 that the charge density profile of lead nucleus will be more diffusive in the presence of the repulsive Yukawa potential, compared to the result from

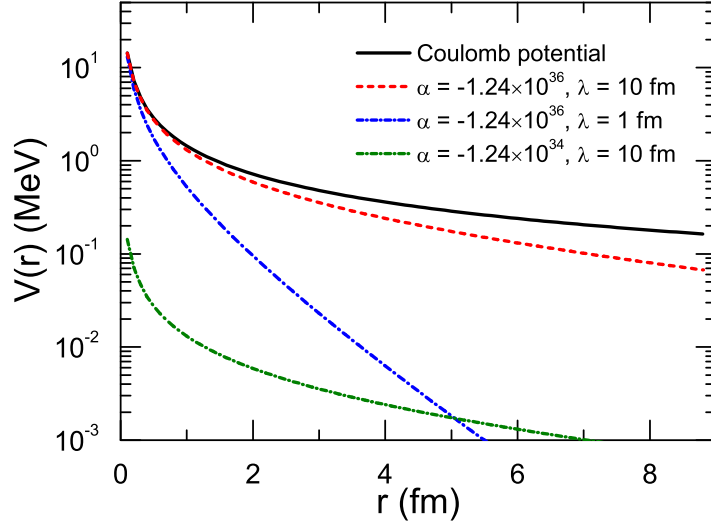


Figure 1. Comparing the Coulomb potential with the non-Newtonian Yukawa potential between two protons of different strength and length scale parameters at different distances of separation. Taken from Ref. [30].

MSL0 force only. Thus, the repulsive Yukawa potential as strong as the Coulomb potential would generally increase the charge radii of finite nuclei, and the effect is larger with a larger λ . This is quite understandable, and it is seen that the effect becomes negligible if the strength of Yukawa potential is 1% of the Coulomb potential.

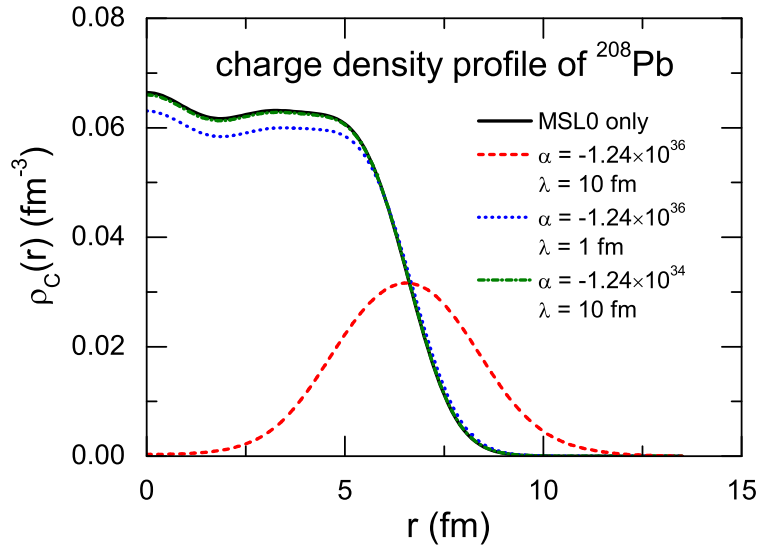


Figure 2. Charge density profiles of lead nucleus from Skyrme-Hartree-Fock calculation with the MSL0 Skymre force only and with an additional non-Newtonian Yukawa potential of different strength and length scale parameters. Taken from Ref. [30].

3.2. Nuclear Constraint on the Non-Newtonian potential

From the above discussion, we intuitively know that the Yukawa-type non-Newtonian gravitational potential will change the finite nuclei properties unless it is of short range and/or much weaker than the Coulomb potential. To avoid modifying the well-determined nucleon-nucleon interaction available, we require that the Yukawa potential will not change much the finite nuclei properties which have been well described by our framework in nuclear physics, at least within the uncertainty from nuclear predictions. Especially, the charge radii and binding energies, which can be very accurately measured in experiments, are mostly related to the well-constrained isoscalar part of the nucleon-nucleon interaction but are not sensitive to the less certain isovector part and/or three-body interaction, and they can now be reproduced by the MSL0 Skyrme force within about 2%. This 2% uncertainty is thus the largest room for the possible existence of the non-Newtonian gravity at femtometer scale.

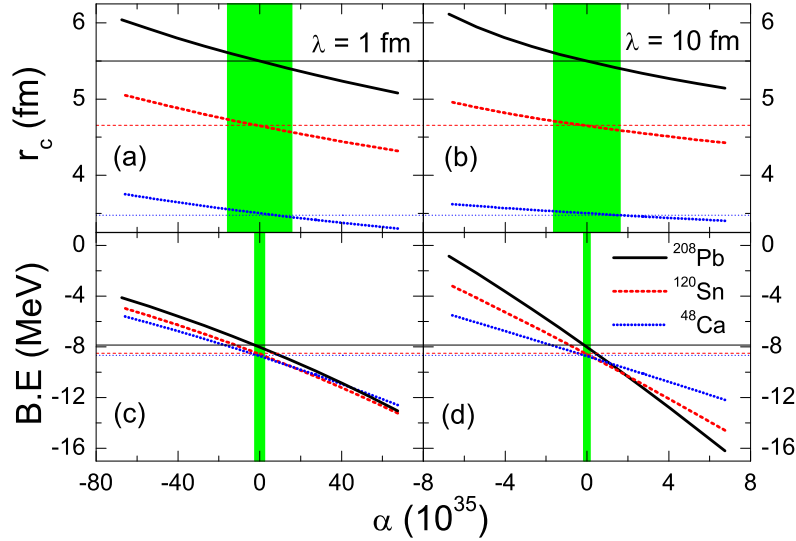


Figure 3. The charge radii (r_c) and binding energies per nucleon (B.E.) of ^{208}Pb , ^{120}Sn , and ^{40}Ca nuclei as functions of the strength parameter α of the non-Newtonian Yukawa potential at length scales $\lambda = 1$ and 10 fm. The horizontal lines are the mean values of the experimental data [31, 32]. $\alpha = 0$ is the case without non-Newtonian potential, and the green band is the largest room for α that the change of the charge radius or binding energy of ^{208}Pb due to the existence of the non-Newtonian potential is within 2%. Taken from Ref. [30].

How the charge radii and binding energies from medium to heavy nuclei change with the strength parameter α of the Yukawa potential at length scales $\lambda = 1$ and 10 fm is displayed in Fig. 3. In the presence of the repulsive (attractive) Yukawa potential, corresponding to a negative (positive) value of α , the charge radii increase (decrease) and the nuclei become less (more) bound, which is quite understandable. The change is almost linear to α and more sensitive to the strength parameter for $\lambda = 10$ fm than for $\lambda = 1$ fm. In addition, the change is larger for heavy nuclei ^{208}Pb due to the finite-range nature of the Yukawa potential. Also plotted in the figure is the 2% uncertainty of both charge radius and binding energy of ^{208}Pb from the MSL0 force only, which are shown with green bands. The green bands are thus the constraints for the strength parameter of the Yukawa potential.

The constraints on the strength of the non-Newtonian potential at femtometer scale from nuclei binding energies and charge radii obtained above are compared with those from other

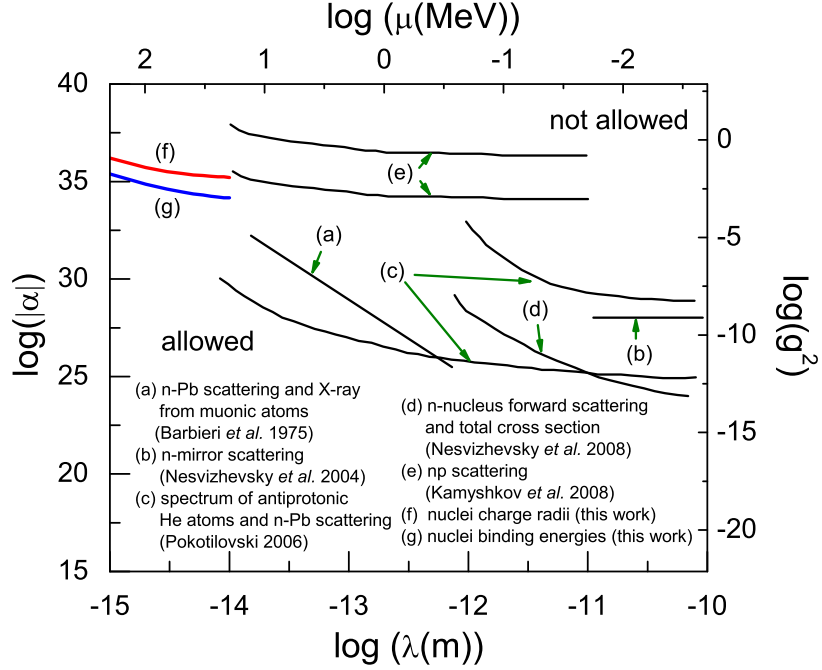


Figure 4. Constraints of the non-Newtonian Yukawa potential at femtometer scale from nuclei binding energies and charge radii in the present work together with those from neutron scattering experiments at larger scales. Taken from Ref. [30].

works mostly using neutron scattering experiments [33, 34, 35, 36, 7] at larger scales in Fig. 4. It is found the upper limit is larger at shorter distances, reflecting the increasing difficulty of the experiments and the stronger potential at work at smaller scale. In addition, it is found that our constraint covers the previously unexplored region and is a smooth expansion of those at larger scales. Typically, the constraint from nuclei binding energy can be written as

$$\log(|\alpha|) < 1.75/[\lambda(\text{fm})]^{0.54} + 33.6, \quad (26)$$

and that from nuclei charge radius can be written as

$$\log(|\alpha|) < 1.18/[\lambda(\text{fm})]^{0.79} + 35.0, \quad (27)$$

for $\lambda = 1 \sim 10$ fm. Since the constraint from the binding energy is stronger, it can be used as the nuclear constraint on the non-Newtonian gravity at femtometer scale, and it also leads to the upper limit of the boson-nucleon coupling constant in the form of

$$\log(g^2) < 0.10[\mu(\text{MeV})]^{0.54} - 3.53, \quad (28)$$

if the boson mass μ is between 20 MeV and 200 MeV.

4. Summary and Outlook

In this talk we report our recent results of nuclear constraints on non-Newtonian gravity at femtometer scale. To do this, we consistently incorporate the Yukawa-type non-Newtonian gravitational potential in the Skyrme-Hartree-Fock calculation. The nucleon interaction is represented by the well-established MSL0 Skyrme force. By assuming that the Yukawa potential

will not change the binding energies and charge radii of finite nuclei by 2%, the strength of the Yukawa potential is constrained within $\log(|\alpha|) < 1.75/[\lambda(\text{fm})]^{0.54} + 33.6$ for $\lambda = 1 \sim 10$ fm, so that the calculated properties of finite nuclei will not be in conflict with the very accurate experimental data available. This constraint may serve as a useful reference in constraining properties of weakly-coupled gauge bosons and further explorations of possible extra dimensions at femtometer scale.

Acknowledgments

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